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How to measure the intercept of the BFKL pomeron at HERA

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Abstract

Determination of the intercept of the BFKL pomeron is one of the pressing issues in the high energy physics. Earlier we have shown that, at the dipole size $r = r_{\Delta} = (0.1 - 0.2)$f, the dipole cross section $\sigma(x, r)$ which is a solution of the generalized BFKL equation, exhibits a precocious asymptotic behavior $\sigma(x, r) \propto \left(\frac{1}{x}\right)^{\Delta_{\mathbf{I\!P}}}$. In this paper we discuss how measuring $F_L(x, Q^2)$ and $\partial F_T(x, Q^2)/\partial \log Q^2$ at $Q^2 = (10 - 40)\mathrm{GeV}^2$ and $Q^2 = (2 - 10)\mathrm{GeV}^2$, respectively, one can probe $\sigma(x, r_{\Delta})$ and directly determine the intercept $\Delta_{\mathbf{I\!P}}$ of the BFKL pomeron in the HERA experiments.

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The intercept $\Delta_{\bf I\!P}$ of the BFKL pomeron [1] is one of the fundamental parametes of QCD. In the BFKL regime, at sufficiently large $\frac{1}{x}$, all the structure functions must have the behaviour

$$F_i(x,Q^2) \propto \left(\frac{1}{x}\right)^{-\Delta_{\bf I\!P}} \tag{1}$$

with the exponent (intercept) $\Delta_{\bf I\!P}$ which is independent of Q^2 (hereafter Q^2 is the virtuality of the photon, x is the Bjorken variable and we refer to deep inelastic scattering (DIS) at very large $\frac{1}{x}$ as the diffractive DIS). Very large $\frac{1}{x} \sim 10^5$ are attainable in the HERA DIS experiments, and it is important to find out whether the pomeron's intercept $\Delta_{\bf I\!P}$ can be measured at HERA or not.

Recently we have examined [2-4] the onset of the BFKL regime in the framework of our s-channel lightcone approach to diffractive DIS [5,6] and our generalized BFKL equation for the dipole cross section [2-4,6,7]. Our conclusion was that the the onset of the BFKL asymptotics as a global feature of DIS is unlikely to happen in the kinematic range of HERA. However, an important observation [2,3] is that the x-dependence of the dipole cross section $\sigma(x,r)$, and which is a solution of our generalized BFKL equation, exhibits for the dipole size $r = r_\Delta = (0.1-0.2)$f, a precocious asymptotic behaviour (1). This dictates the obvious strategy: in order to expertimentally determine the pomeron's intercept $\Delta_{\bf I\!P}$, one must find observables which are dominated by $\sigma(x,r)$ at $r \sim r_\Delta$. An example of such an observable - the real and virtual photoproduction of the open charm at $Q^2 \lesssim 10\mathrm{GeV}^2$ - has already been discussed in [2,3].

In this paper we demonstrate that the longitudinal structure function $F_L(x,Q^2)$ and the slope of the transverse structure function $\partial F_T(x,Q^2)/\partial \log Q^2$ emerge as local probes of the dipole cross section at $r^2 \approx B_{T,L}/Q^2$ with $B_T \approx 2.3$ and $B_L \approx 11$. Therefore, in order to measure the pomeron's intercept $\Delta_{\bf I\!P}$, one must concentrate on the x-dependence of $F_L(x,Q^2)$ at

$$Q^2 = \frac{B_L}{r_\Delta^2} \approx (10-40)\mathrm{GeV}^2 \tag{2}$$

and of $\partial F_T(x,Q^2)/\partial \log Q^2$ at

$$Q^2 = \frac{B_T}{r_\Delta^2} \approx (2-10)\mathrm{GeV}^2\,. \tag{3}$$

This range of Q^2 is easily accessible at HERA and, as a matter of fact, falls in the region of highest counting rates, which makes the measurement of $\Delta_{\bf I\!P}$ possible already in the near future.

The starting point of derivation of the above results is the s-channel lightcone approach to diffractive DIS [4,5]. Here the photoabsorption cross section for the (T) transverse and (L) longitudinal photons can be written down as

$$\sigma_{T,L}(\gamma^* N, x, Q^2) = \int_0^1 dz \int d^2\vec{r}\, |\Psi_{T,L}(z,r)|^2 \sigma(x,r) \,, \tag{4}$$

where $\sigma(x,r)$ is the total cross section of interaction of the $q\bar{q}$ colour dipole of transverse size r with the nucleon target, and is a solution of the generalized BFKL equation [2-4,6,7]. The wave functions of the $q\bar{q}$ Fock states of the photon were derived in [5] and read

$$|\Psi_T(z,r)|^2 = \sum e_f^2 |\Psi_T^{(f\bar{f})}(z,r)|^2 = \frac{6\alpha_{em}}{(2\pi)^2} \sum_1^{N_f} Z_f^2 \{[z^2 + (1-z)^2]\varepsilon^2 K_1(\varepsilon r)^2 + m_f^2 K_0(\varepsilon r)^2\} \,, \tag{5}$$

$$|\Psi_L(z,r)|^2 = \sum e_f^2 |\Psi_T^{(f\bar{f})}(z,r)|^2 = \frac{6\alpha_{em}}{(2\pi)^2} \sum_1^{N_f} 4 Z_f^2\, Q^2\, z^2 (1-z)^2 K_0(\varepsilon r)^2 \,, \tag{6}$$

where $K_\nu(x)$ are the modified Bessel functions, $\varepsilon^2 = z(1-z)Q^2 + m_f^2$, m_f is the quark mass and z is the fraction of photon's light-cone momentum q_- carried by one of the quarks of the pair $(0 < z < 1)$. The flavour and Q^2 dependence of structure functions is concentrated in wave functions (5,6), whereas the dipole cross section $\sigma(x,r)$ is universal for all flavours. We emphasize that the factorization of the integrands in Eq. (4) follows from an *exact* diagonalization of the diffraction scattering matrix in the $(\vec{r}, z)$-representation [5,6]. Furthermore, the dipole-cross section representation (4) and wave functions (5,6) are valid in the BFKL regime, i.e., also beyond the Leading-LogQ^2 approximation (LLQA). The x (energy) dependence of the dipole cross section $\sigma(x,r)$ comes from the higher $q\bar{q}g_1...g_n$ Fock states of the photon, i.e., from the QCD evolution effects [6] described by the generalized BFKL equation. The transverse and longitudinal structure functions are given by the familar equation $F_{T,L}(x,Q^2) = (Q^2/4\pi\alpha_{em})\sigma_{T,L}(x,Q^2)$. We advocate using $F_T(x,Q^2) = 2xF_1(x,Q^2)$, because it has simpler interpretation than $F_2 = F_T + F_L$, which mixes interactions of the transverse and longitudinal photons.

The ratio $\sigma(x,r)/r^2$ is a smooth function of r, and it is convenient to use the representation

$$F_T(x,Q^2) = \frac{1}{\pi^3} \int \frac{dr^2}{r^2} \frac{\sigma(x,r)}{r^2} \sum e_f^2 \Phi_T^{(f\bar{f})}(Q^2, r^2) \,, \tag{7}$$

$$F_L(x,Q^2) = \frac{1}{\pi^3} \int \frac{dr^2}{r^2} \frac{\sigma(x,r)}{r^2} \sum e_f^2 W_L^{(f\bar{f})}(Q^2, r^2) \,, \tag{8}$$

$$\frac{\partial F_T(x,Q^2)}{\partial \log Q^2} = \frac{1}{\pi^3} \int \frac{dr^2}{r^2} \frac{\sigma(x,r)}{r^2} \sum e_f^2 W_T^{(f\bar{f})}(Q^2, r^2) \,, \tag{9}$$

where the weight functions $\Phi_T^{(f\bar{f})}$ and $W_{T,L}^{(f\bar{f})}$ are defined by

$$\Phi_T^{(f\bar{f})}(Q^2,r^2) = (\pi^2/4\alpha_{em})\int_0^1 dz\, Q^2 r^4 |\Psi_T^{(f\bar{f})}(z,r)|^2 \,, \tag{10}$$

$$W_L^{(f\bar{f})}(Q^2,r^2) = (\pi^2/4\alpha_{em})\int_0^1 dz\, Q^2 r^4 |\Psi_L^{(f\bar{f})}(z,r)|^2 \tag{11}$$

$$W_T^{(f\bar{f})}(Q^2,r^2) = \frac{\partial \Phi_T^{(f\bar{f})}(Q^2,r^2)}{\partial \log Q^2} \,. \tag{12}$$

These weight functions ar shown in Figs. 1,2. Following the above outlined strategy, we wish identify the observable $f(x)$ which is dominated by the contribution from $r = r_\Delta$, so that its x-dependence can be used to determine the pomeron's intercept $\Delta_{\mathbf{IP}} = -\log f(x)/d\log x$. Fig. 1 shows that the transverse structure function $F_T(x,Q^2)$ receives contributions from $B_T/Q^2 \lesssim r^2 \lesssim 1/m_f^2$, does not zoom at $r \sim r_\Delta$, and as such it is not suited for determination of $\Delta_{\mathbf{IP}}$.

Let us discuss the salient features of $\Phi_T^{(f\bar{f})}(Q^2,r^2)$ in more detail. At large Q^2 it developes a plateau of unit height. The width of the plateau rises $\propto \log Q^2$, and the emergence of this plateau signals the onset of the familiar LLQA. At $r^2 \gg B_T/Q^2$, the weight function $\Phi_T^{(f\bar{f})}(Q^2,r^2)$ is a scaling function of Q^2 and, because $\sigma(x,r)$ does not depend on Q^2, the Q^2-dependence of $F_T(x,Q^2)$ entirely comes from the expansion of the plateau of $\Phi_T^{(f\bar{f})}(Q^2,r^2)$ towards small r with increasing Q^2.

Notice a very slow onset of the scaling regime for the charmed quark contribution. The corresponding $\Phi_T^{(f\bar{f})}(Q^2,r^2)$ starts developing a plateau only at very large Q^2. Fig. 1 strongly suggests that the charmed quarks can be treated as massles partons, the charm can be regarded an active flavour and the LLQA becomes accurate for the charmed quarks, only at $Q^2 \gtrsim 100\text{GeV}^2 \sim 40m_c^2$. At very small sizes, $Q^2r^2 \ll B_T$ and $r^2m_c^2 \ll 1$, the weight functions for the charmed and light quarks converge to each other already at moderate Q^2. For the light u,d quarks, the curves shown are for $m_{u,d} = 0.15\text{GeV}$. In this case the onset of the plateau, and of the LLQA thereof, requires $Q^2 \gtrsim (2-3)\text{GeV}^2$. (With the more conservative $m_{u,d} = 0.3\text{GeV}$ the plateau only starts developing, and the LLQA only will be accurate, at $Q^2 \gtrsim 10\text{GeV}^2$). In Fig. 1 we also show

$$\Phi_T(Q^2,r^2) = \frac{9}{11}\sum_{u,d,s,c,b} e_f^2 \Phi_T^{(f\bar{f})}(Q^2,r^2) \,,$$

which would have been equal to unity if all 5 flavours were active. The large mass of the charmed quark has a very profound effect, and it is evidently premature to speak of the 4 active flavours unless $Q^2 \gtrsim (100-200)\text{GeV}^2$.

Evidently, $W_T^{(f\bar{f})}(Q^2,r^2) = \partial\Phi_T^{(f\bar{f})}(Q^2,r^2)/\partial\log Q^2$ will be a sharply peaked function of r^2, which only is nonvanishing at $r^2 \sim 1/Q^2$. In Fig. 2 we show $W_{T,L}^{(f\bar{f})}(Q^2,r^2)$ as a function of the natural variable $\log(Q^2r^2)$. At large Q^2 and $Q^2r^2 \sim 1$, $W_T^{(f\bar{f})}(Q^2,r^2)$ becomes a scaling function of Q^2, in agreement with the conventional expectation that in DIS the only relevant scale is $1/\sqrt{Q^2}$. At the asymptotically large Q^2, the charmed and light quarks have identical $W_T(Q^2,r^2)$. The Q^2, r^2 and the flavour dependence of $W_L(Q^2,r^2)$ resembles that of $W_T(Q^2,r^2)$, apart from the strikingly slower onset of the asymptotic scaling form of $W_L(Q^2,r^2)$ and, consequently, of the LLQA for the longitudinal structure function.

The weight functions $W_{T,L}(Q^2,r^2)$ have their center of gravity at $Q^2r^2 = B_{T,L}$, where $B_T \approx$ 2.1, 2.3, 2.6, 2.8 and $B_L \approx$ 8.3, 10.2, 10.6, 12 at $Q^2 = 0.75$, 4.5, 30 and 480GeV^2, respectively. Therefore, at large Q^2 (summation only over active flavours is understood)

$$F_L(x,Q^2) = \frac{1}{\pi^3}\sum e_f^2 \left.\frac{\sigma(x,r)}{r^2}\right|_{r^2=B_L/Q^2}, \tag{13}$$

$$\frac{\partial F_T(x,Q^2)}{\partial\log Q^2} = \frac{1}{\pi^3}\sum e_f^2 \left.\frac{\sigma(x,r)}{r^2}\right|_{r^2=B_T/Q^2}, \tag{14}$$

and the x dependence of $F_L(x,Q^2)$ and $\partial F_T(x,Q^2)/\partial\log Q^2$ at values of Q^2 given by equations (2) and (3), respectively, follows a precocious asymptotic behaviour of $\sigma(x,r_\Delta)$ and measures the pomeron's intercept.

Fig. 1 shows that, by the numerical coincidence $r_\Delta \sim 1/m_c$, at $Q^2 \lesssim 10$GeV^2 the charm contribution to $F_T(x,Q^2)$ is also dominated by the contribution from $r \sim r_\Delta$. Therefore, as we suggested in [2], the measurement of the x-dependence of the charm structure function $F_T^{(c\bar{c})}(x,Q^2)$ also allows determination of $\Delta_{\mathbf{I\!P}}$. The availability of the above three methods of determination of the pomeron's intercept is very important as allows the consistency checks.

Two important points, common to all the above methods of determination of $\Delta_{\mathbf{I\!P}}$, must be emphasized. Firstly, they are quite insensitive to the belated onset of LLQA and to the precise number of active flavours (ss a matter of fact, the excitation of charm at $Q^2 \lesssim 10$GeV^2 corresponds to a deeply sub-LLQA regime). The suggested technique only depends on the position of the peak of $W_{T,L}(Q^2,r^2)$, the existence of this peak does not require the applicability of LLQA. Secondly, it would have been somewhat more accurate to substitute $\sigma(x,r)$ in the integrand of (4) for $\sigma(\frac{x}{z},r)$. However, as far as we zoom at $r \approx r_\Delta$, at which $\sigma(x,r_\Delta) \propto \left(\frac{1}{x}\right)^{\Delta_{\mathbf{I\!P}}}$,

this substitution does not affect the x-dependence in Eqs. (13,14) and the proposed determination of the pomeron's intercept $\Delta_{\bf IP}$. With the advent of the high precision data, the above approach can readily be extended to include the effects of the z-dependence of the effective energy in the dipole cross section $\sigma(x,r)$, and also the effects of the z-r correlations in the corresponding wave functions.

The comment on uncertainties and challenges is in order. Much depends on the specific value of r_Δ. Our analysis [2,3] has shown that $r_\Delta \sim \frac{1}{2}R_c$, where R_c is the correlation radius for the perturbative gluons. This correlation radius was a subject of active studies in the lattice QCD, which suggest $R_c \approx 0.3$f (for the review and references see [8]). The analysis [2,4,6] also strongly suggests that the existence of the magic dipole size r_Δ at which the dipole cross section exhibits a precocious asymptotic behaviour, is quite a generic property of the BFKL equation. However, besides the gluon correlation radius R_c, there may be other nonperturbative parameters which can affect both the pomeron's intercept and the magic size r_Δ. Here we we only wish to notice that, if $R_c \sim 0.3$f as the lattice QCD studies suggest, then $r_\Delta \sim \frac{1}{2}R_c$ corresponds to distances at which the QCD coupling is relatively small. Also, the analysis [4] suggests that at so small a value of r_Δ the nonperturbative component of the dipole cross section can be neglected. For this reasons, one can anticipate weak dependence of r_Δ on the nonperturbative effects at $r \gtrsim 1$f. More studies on the nonperturbative effects in the BFKL eqation are evidently needed.

There are two byproducts of the above analysis, both of potential significance for determination [9,10] of the gluon structure function $G(x,Q^2)$ from $F_L(x,Q^2)$ and $\partial F_T(x,Q^2)/\partial \log Q^2$: a very slow onset of the scaling regime for the charmed quark contribution to structure functions and the subtantial difference between B_T and B_L. The former is important for the number of active favours, the latter suggests that $F_L(x,Q^2)$ and $\partial F_T(x,Q^2)/\partial \log Q^2$ will give $G(x,q^2)$ at different values of q^2, which differ by a large factor B_L/B_T, and which are both different from Q^2. The anomalously large value of B_L suggests a very slow onset of the short-distance dominance, and the potentially large corrections to LLQA, for the longitudinal structure function. The very slow onset of LLQA for charmed quarks affects a number of active flavours and is still another potential source of large corrections to the much discussed LLQA relations [9,10] between $G(x,Q^2)$ and $F_L(x,Q^2)$, $\partial F_T(x,Q^2)/\partial \log Q^2$. The detailed discussion of these corrections

to LLQA will be presented elsewhere [11].

Conclusions:

The purpose of this study has been an analysis of determination of the pomeron's intercept $\Delta_{\mathbf{IP}}$ from measurements of the longitudinal $F_L(x,Q^2)$, and the slope $\partial F_T(x,Q^2)/\partial \log Q^2$ of the transverse, structure functions at HERA. Our principal finding is that $\Delta_{\mathbf{IP}}$ can readily be determined at HERA concentrating on the special range of Q^2 given by Eqs. (2,3). The x-dependence of the excitation of charm at $Q^2 \lesssim 10\text{GeV}^2$ provides the third, independent, determination of $\Delta_{\mathbf{IP}}$. The important virtue of proposed methods is their model-independence: they require neither a validity of the Leading-LogQ^2 approximation, nor a knowledge of the number of active flavours. They require, though, a knowledge of the magic dipole size r_Δ, which we related to the nonperturbative correlation radius of the perturbative gluons, known from the lattice QCD studies. Still, in parallel with the anticipated experimental determinations of $\Delta_{\mathbf{IP}}$, more work on the noneprturbative effects in the BFKL equation is needed.

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Figure captions

Fig.1 - The weight function $\Phi_T(Q^2, r^2$: (a) for the light flavours u and d, (b) for the charmed quark, (c) for the b-quark, (d) the global weight function for 5 flavours (u, d, s, c, b). The dashed, dotted, solid, lohg-dashed and dot-dashed curves are for $Q^2 = 0.75,\ 4.5,\ 30,\ 240,$ $2000\mathrm{GeV}^2$, respectively.

Fig.2 - The weight functions $W_{T,L}(Q^2, r^2)$: (a) the light flavours (u, d), (b) the charmed quark, (c) the global weight function for 5 flavours (u, d, s, s, b). The dashed, dotted, dot-dashed and solid curves are for $Q^2 = 0.75$, 4.5, 30, $240\mathrm{GeV}^2$, respectively.